AN INVESTIGATION OF LOW LAMBDA YIELD FAILURE DESIGN OF STIFFENED CYLINDRICAL PRESSURE VESSELS UNDER EXTERNAL HYDROSTATIC PRESSURE

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by

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ABSTRACT

The designer of stiffened cylindrical pressure vessels is often faced with the problem of comparing structures of wiedly varying geometries. To do this, he makes use of a number of dimensionless parameters whose function is to reduce the number of variables in the design problem.

The object of this thesis was to investigate the effect of the dimensionless parameter, λ, on stiffened cylindrical pressure vessel design. In particular, the λ region where collapse of vessels due to yield-initiated buckling between the frames was identified and delimited. This region was used as the first iteration in a design procedure aimed at producing a vessel of maximum structural efficiency. Design curves which may be used to develop such a vessel are provided for shell thicknesses up to $h/D_m = .01$. These curves are based on both maximum principle stress and Hencky-von Mises stress criteria and provision is made in a computer program for use of other methods. A modified Foppl Formula is used for frame evaluations.

The formulas used range from conservative to liberal in their approximations of collapse pressures. However, use of even the most conservative formulas results in structural efficiencies which compare favorably with present-day practices.

Thesis Supervisor: John Harvey Evans
Title: Professor of Naval Architecture

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NOMENCLATURE

A	Cross sectional area of frame, in^2
A_b	Cross sectional area of frame plus plating of width b, in^2
A_1, A_2, A_3	Plasticity coefficients, see Reference (10)
b	Width of frame at shell, in
c	Distance from neutral axis of bending to extreme outside fiber, in
D	Outside diameter of shell, in
d	Depth of rectangular frame, in
D_b	Diameter to neutral axis of frame plus plating of width b, in
D_m	Diameter to neutral axis of shell alone, in
E	Modulus of elasticity, lbs/in^2
e	Eccentricity, in
E_s	Secant modulus of elasticity, lbs/in^2
f	Width of flange of T frame, in
h	Shell thickness, in
I_b	Moment of inertia, frame plus plating of width b, in^4
I_e	Moment of inertia, frame plus plating of effective width, see Reference (10), in^4
I_f	Moment of inertia, frame alone, in^4
I_1	Moment of inertia, frame plus plating of width L, in^4
L	Unsupported span of plating, in
L_b	Bulkhead spacing, in
L_f	Frame spacing, in
L_1	Unsupported span of plating taken as frame sapcing, in

n Number of complete waves in buckled configuration - circumferentially

p_c Collapse pressure, lbs/in^2

p_l Actual collapse pressure desired, lbs/in^2

p_{c1}, p_{c2}, p_{c3} Temporary values of p_c for iteration

q_f Collapse load per unit axial length of a ring frame, lbs/in^2

R Outside shell radius, in

R_f Radius to neutral axis of frame, in

R_m Radius to neutral axis of shell, in

R_l Radius to neutral axis of frame plus shell of width L, in

V Volume displaced by cylinder per unit length, in^3/ft

Wt Weight per unit length, lbs/ft

Δ Weight of salt water displaced by cylinder per unit length, lbs/ft

η Efficiency - defined in Section III

λ Slenderness Ratio - defined in Section I

μ Poissons ratio

σ_F Axial stress at mid-thickness, lbs/in^2

σ_T Tangential (circumferential) stress at mid-thickness, lbs/in^2

σ_y Material yield stress, lbs/in^2

ψ Pressure factor - defined in Section I

I. INTRODUCTION

Stiffened cylindrical pressure vessels under external hydrostatic pressure fail by one or any combination of three collapse modes. While both theoretical and empirical expressions have been developed to determine collapse pressures for all modes, it is up to the designer to determine what structural proportions will result in the most efficient use of material strength and stiffness properties for a required collapse pressure. Two dimensionless parameters which have been widely used to define different geometries and their capabilities are ψ and λ. ψ is called pressure factor and it is a ratio of a structure's actual collapse pressure to the collapse pressure of an unstiffened tube with the same h/D_m ratio.

$$\psi = \frac{p_c}{2\frac{h}{D_m}\sigma_y}$$

λ is called slenderness ratio. It is analogous to the slenderness ratio, l/r, of column theory, however, it includes terms which allow for different material properties.

$$\lambda = \sqrt[4]{\frac{(L/D_m)^2}{(h/D_m)^3}} \sqrt{\frac{\sigma_y}{E}}$$

Despite their fairly wide use by designers in comparisons of different structural geometries, this

researcher is unaware that any attempt has been made to develop a relationship between these parameters which would indicate their effect on structural efficiency. Also, it has been stated that these parameters are sometimes misleading (7), however, no definite limits on their usefulness have been established in the literature, and only a few other dimensionless parameters have been developed to be used instead. A need exists therefore, to define the limits of usefulness of both ψ and λ and to develop within those limits, relationships between them and structural efficiency which may be readily applied in the early design stages of a stiffened cylindrical shell.

In this report, two failure criteria, maximum circumferential stress and Hencky-von Mises, have been used to develop relationships between λ, ψ, and efficiency. In the course of the development, certain trends of the parameters have been noted and in the concluding remarks, useful limits of λ and ψ are indicated. In particular, the "low lambda" region has been defined and an introduction made to the concept of maximizing structural efficiency within that region. Design curves are presented to enable optimization within the low lambda region of structures with collapse pressures up to 2500 psi and examples of the process are given. Special note should be taken

of Appendices B and C where remarks concerning very low lambda design are made.

II. DEVELOPMENT OF DESIGN CRITERIA

Historically, the largest and most critically designed stiffened cylindrical pressure vessels have been intended for use as submersibles. Such vessels, under external hydrostatic pressure, have been known to collapse in one, or a combination of three modes. Since the mechanism which precipitates failure in each mode is different, the designer must take all three modes into consideration when designing for a given collapse pressure. However, he has a choice in the design method he may use when developing a highly efficient structure. He may design toward one mode of collapse, seeking to eliminate the possibility of collapse in others while changing structural proportions to obtain a high efficiency. His alternative is to simply design toward the most efficient structure, not worrying about the mode of collapse as long as the required collapse pressure is obtained.

An example of the first method may be found in the design of the relatively shallow-diving submarines of World War II. These vessels were designed to collapse by asymmetric buckling between frames. It was felt that imminent collapse would be signalled by the appearance of lobes girdling the hull and the crew would be given a chance to surface before total collapse occurred (22).

This explanation indicates that for the volume-limited submarines of that era, efficiency criteria could be established to a large degree by safety considerations.

For deeper diving, weight-limited submarines, the first method proved inadequate and the second method was refined in a number of ways to allow development of highly efficient vessels. A refinement which will now be discussed is one which assumed that ψ is limited to 1.0. This assumption used the maximum circumferential stress criterion and minimized weight by using the smallest possible number of frames. It resulted in a structure proportioned such that $\lambda = .8$. In order to visualize this approach, figure 1 has been developed.

Figure 1 is a plot of ψ versus d/h (frame depth/ shell thickness) for four λ values. As implied by the abscissa scale which indicates relative frame strength, the frames are rectangular and only their depth is varied to vary strength. The basic geometries used in this figure are taken from a report by Trilling (19) and one of his curves (curve 2), as well as his experimental data (crosses) are plotted. The other three curves plotted on figure 1 represent the effect of changing λ. In all three cases, the distance between frames was the only parameter varied. Values of ψ were obtained by using

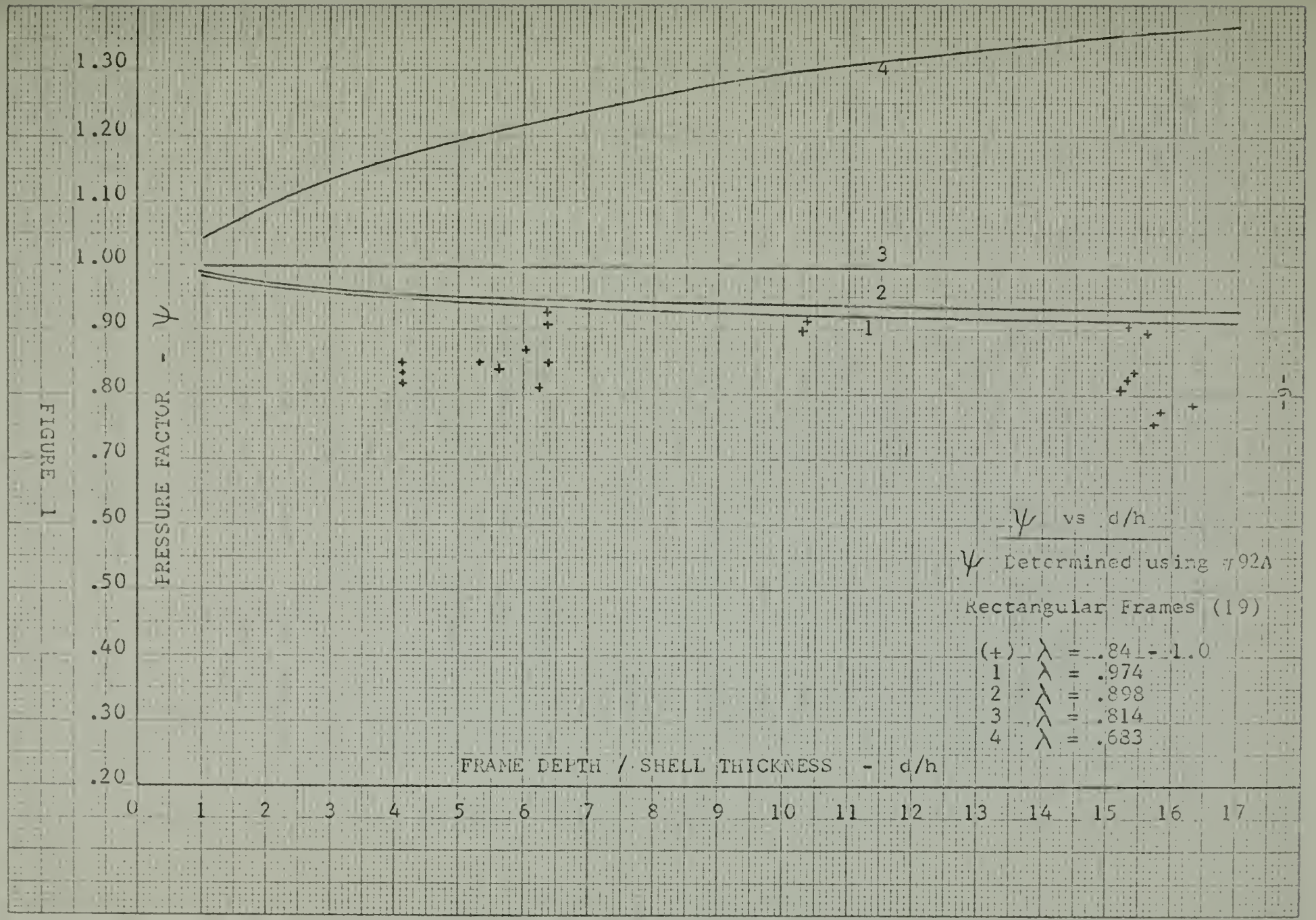

1.30
1.20
1.10
1.00
.90
.80
.70
.60
.50
.40
.30
.20
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
PRESSURE FACTOR - ψ
FRAME DEPTH / SHELL THICKNESS - d/h
4
3
2
1
ψ vs d/h
ψ Determined using #92A
Rectangular Frames (19)
(+) λ = .84 - 1.0
1 λ = .974
2 λ = .898
3 λ = .814
4 λ = .683

FIGURE 1

von Sanden and Gunther's formula #92A to predict collapse pressure.

It will be noted that curve 3 of this figure lies almost entirely along the line, $\psi = 1.0$. The λ value of this curve is .814, and it is this relationship which forms the basis of the design method cited above. Just as the earlier method of designing for a particular collapse mode proved inadequate, this method has also been unable to develop the efficient structures necessary to resist high pressures. The remainder of this section will be devoted to developing a design method which will result in highly efficient structures. To begin this development, a further look at figure 1 is necessary.

On the basis of curve 2 and the experimental points, one of Trilling's conclusions was that "except for very short thick tubes, . . . according to formula #92A the stronger the frame, the weaker the vessel." This was explained by noting that very strong frames form "hard spots" when, under external pressure, they contract less than the shell. The hard spots, in turn, cause stress concentrations in the otherwise uniformly contracting shell and lower collapse pressures.

Curve 4 indicates, however, that this conclusion is

not necessarily true, even for thin shells and large frames. The only difference between the model of curve 4 and Trilling's models is the decreased frame spacing which results in different λ values; .898 for Tilling's models (curve 2) and .683 for curve 4. According to curve 4, shell strength actually increases with frame size when the frames are relatively close together. The seeming contradiction of Trilling may be explained by an extension of his logic. If the frames causing stress concentrations are closer together (low lambda) the overall stress pattern will be uniform enough that premature yielding and buckling will not take place.

The obvious lower limit of λ is obtained when the frames are in contact ($\lambda \rightarrow 0$) (24) thereby creating a thicker shell (rectangular frames) or a webbed sandwich-type construction (T-frames). Both of these geometries result in putting the frame-caused stress concentrations in such close proximity that the overall stress pattern approaches uniformity. This uniformity of stress pattern implies that the yield strength of most of the material in the structure will be utilized before collapse occurs, which, in turn, implies that a structure of very high efficiency is possible in this λ area.

With a lower limit of λ established at 0, an upper

limit is necessary to define the range of geometries where the effect of increasing frame strength is not detrimental to collapse pressure. This limit will be referred to as λ_m. For structures with λ values above λ_m, increasing the frame strength weakens the vessel. For λ values below λ_m, increasing the frame strength strenthens the vessel. An initial value of this limit can be found by direct use of Windenburg's approximation of von Mises' formula for collapse between frames due to elastic shell instability. The development of this formula is outlined in Appendix A. In its simplest form, this formula is:

$$\psi = \frac{1.3}{\lambda^2}$$

Recalling that $\psi = \frac{p_c}{2\frac{h}{D_m}\sigma_y}$, Windenburg's formula determines instability collapse in terms of the hoop stress relation. i.e.,

$$p_c = 2\frac{h}{D_m}\sigma_y \frac{1.3}{\lambda^2}$$

therfore, when $\frac{1.3}{\lambda^2} = 1.0$, failure is due primarily to yielding of the material rather than plastic instability. At this point, $\lambda = 1.14$. If λ 1.14, then instability collapse is indicated since ψ 1.0. In this situation,

strengthening the frames cannot delay collapse. It may be tentatively assumed that when λ 1.14, strengthening the frames increases overall strength. If this is so, then an initial approximation of λ_m is 1.14. It is important to note that this is a very approximate method for determining λ_m. A glance of Trilling's results on figure 1 will indicate that collapse at ψ values below 1.0 has occurred in structures with λ values as low as .89. Also, with this method, λ_m is independent of any individual structure parameters. Since this is questionable, another means of determining λ_m is necessary.

Returning to curve 3 on figure 1, it will be recalled that for the λ value used in that curve, $\psi = 1.0$ for all frame sizes. As long as the frames are of adequate strength or stiffness to carry a given percentage of the shell load, their size has no bearing on the strength of the cylinder. Figure 2 elaborates on this point. The curves of figure 2 are collapse pressures, determined by #92A, plotted as ψ against values of λ. Each curve represents one size of frame and all structures have the same h/D ratio, so that only the distance between frames has been varied to obtain changing λ values. The point at which all curves cross is at $\psi = 1.0$ and represents the afore-mentioned λ_m. Unlike the approximate Windenburg determination of this point, in this determination, λ_m

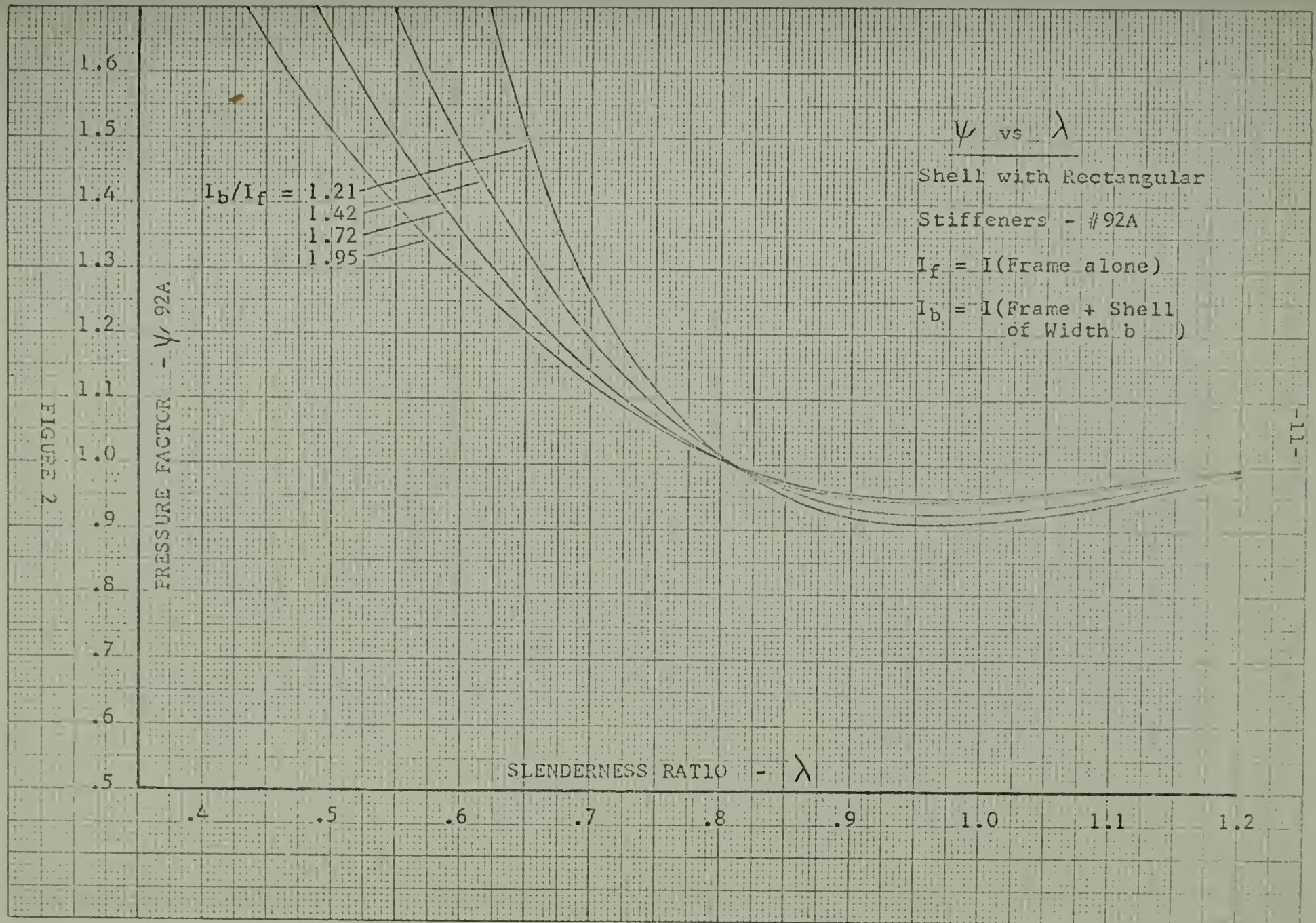

ψ vs λ
Shell with Rectangular Stiffeners - #92A
I_f = I(Frame alone)
I_b = I(Frame + Shell of Width b)
I_b/I_f = 1.21
1.42
1.72
1.95
PRESSURE FACTOR - ψ 92A
SLENDERNESS RATIO - λ
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2

FIGURE 2

depends on the value of h/D as well as the fact that $\psi = 1$. This dependence is borne out in the derivation of an expression for λ_m from formula #92A. This derivation is found in Appendix E and results in the equation:

$$\lambda_m = \sqrt{\frac{2.055 D_m}{h} \frac{\sigma_y}{E}} \qquad (A)$$

which was used to plot h/D_m versus λ_m for different values of σ_y on figure 3.

Figure 3 may be used to determine for a given h/D_m and σ_y what λ is necessary to prevent collapse between the frames due to instability ($\psi < 1.0$). Areas to the right of the curve in question are areas of instability collapse, those to the left, primarily yield failure. As an example of this, Trilling's experimental results have again been plotted. Due to slightly varying geometries and σ_y's, all the models he used fall within the areas of the dots shown. The figure indicates that for σ_y = 30,000 psi (see appendix) all the models failed at a ψ below 1.0. Reference to figure 1 shows that this was indeed the case,and a further glance at Reference (19) indicates that a number of Trilling's models failed by lobar buckling or general instability, rather than by yield initiated collapse.

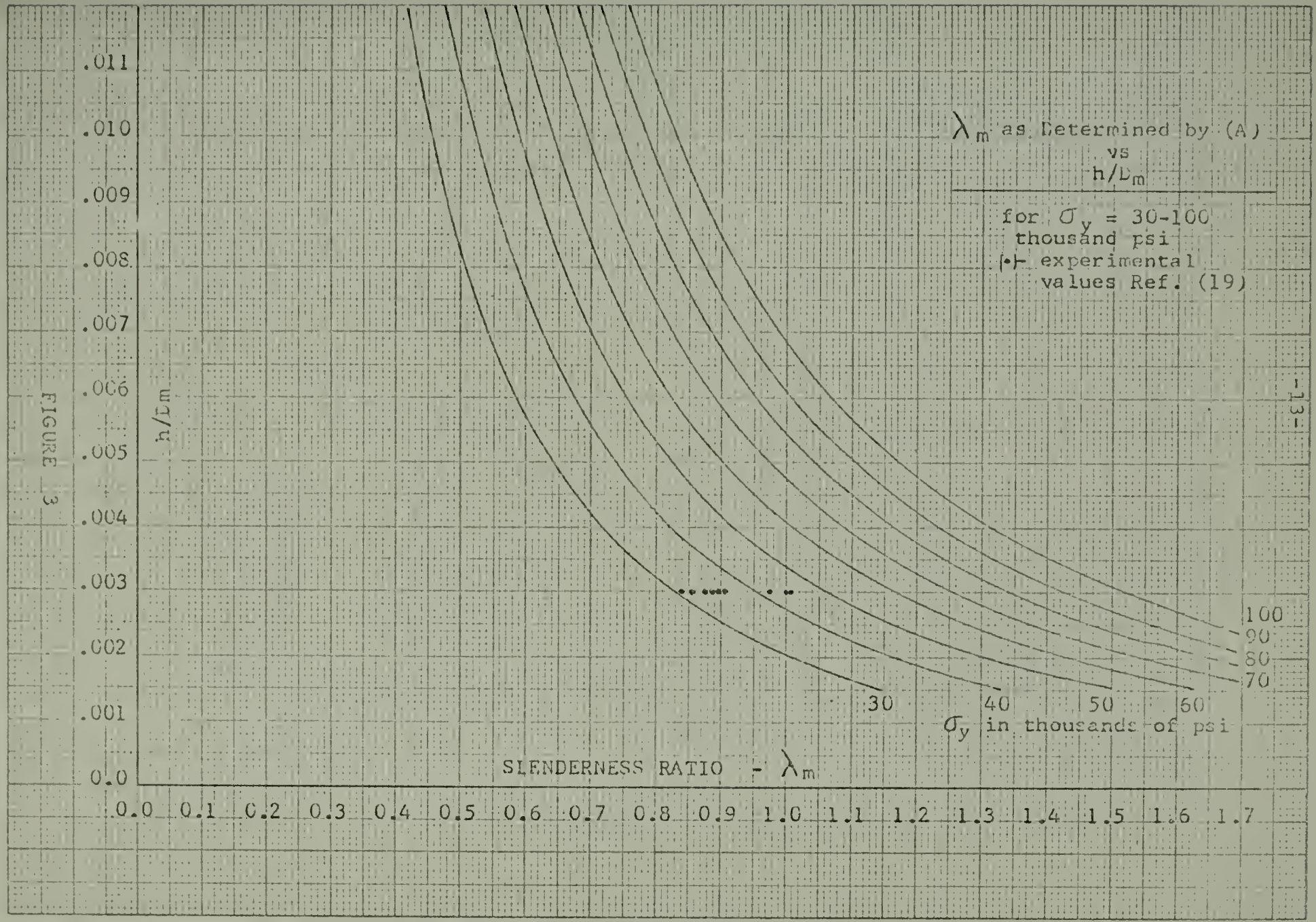

λ_m as Determined by (A)
vs
h/D_m
for σ_y = 30-100 thousand psi
experimental values Ref. (19)
100
90
80
70
60
50
40
30
σ_y in thousands of psi
h/r_m
SLENDERNESS RATIO - λ_m
.011
.010
.009
.008
.007
.006
.005
.004
.003
.002
.001
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7

FIGURE 3

In Appendix A, a number of formulas and yield criterion are cited as being more accurate than the maximum circumferential stress criterion in determining collapse pressure due to yielding. One of these, Wenk's formula, has been investigated using the same models used in figures 1-3 in order to ascertain the influence of increased accuracy on the computation of λ_m. Figure 4 is the result of this investigation.

Wenk applied formulas #92 and #92A to the Hencky-von Mises yield criterion at mid-thickness, mid-bay, and obtained collapse pressures within nine percent of actual experimental values. The Hencky-von Mises criterion in two dimensions is:

$$\sigma_y^2 = \sigma_T^2 - \sigma_T\sigma_F + \sigma_F^2$$

When it is applied to an unstiffened shell where $\sigma_T = 2\sigma_F$, yield failure occurs at $\sigma_T = 1.16\sigma_y$, i.e., $\psi = 1.16$. Just as figure 2 indicated that for the maximum circumferential stress criterion, there is a λ value where frame size has no effect on cylinder strength, figure 4 shows that the same is true for the Hencky-von Mises criterion. However, the λ value in this case occurs when $\psi = 1.16$ instead of 1.0. Wenk's formula:

in the range of low λ values. However, its inclusion of design curves to determine cylinder plastic reserve strength has made it more difficult to apply in computer-based design methods.

It is now generally accepted that stiffened cylindrical shells fail by one of three distinct modes or any combination of them. These modes are (14):

Mode 1. Axisymmetric collapse of the shell between adjacent ring frames.

Mode 2. Asymmetric collapse of the shell between adjacent ring frames (lobar).

Mode 3. Overall asymmetric collapse of the shell and frames together.

Mode one had traditionally been referred to as axisymmetric shell yielding (9,22,23). It has been pointed out, however, that this mode is the result of a combination of yielding and axisymmetric buckling (14,22) and the main difference between this mode and mode two is the difference in resulting collapse patterns, not basic collapse mechanisms. Axisymmetric collapse pressures have been calculated with good accuracy through use of yield criterion alone though, and it is recognized that pre-collapse stresses, as determined by von Sanden and

Gunter (20) or Salerno and Pulos (17), are the primary cause of failure in this mode.

The problem of axisymmetric collapse by elastic buckling has been solved by Salerno and Pulos in (13). Lunchick (10) has investigated axisymmetric failure by inelastic buckling and derived the following expression for it:

$$P_{cr} = \left[\frac{2}{(1-\nu^2)} \left(A_2 - \frac{A_{12}^2}{A_1} \right) C^2 E_s \left(\frac{h}{R} \right)^2 + \frac{1}{6(1-\nu^2)} \frac{A_1}{C^2} E_s \left(\frac{h}{R} \right)^2 \right] \qquad C = \frac{L^2}{\pi^2 R h}$$

von Mises' and Windenburg's formulas for the calculation of failure pressure by elastic asymmetric (lobar) buckling between ring frames have been given. A more recent solution to this problem has been found by Reynolds and is outlined in reference (14). Reynolds has also developed a formula for the inelastic buckling of stiffened cylindrical shells which appears to be reasonably accurate, but is consistently high in its calculation of collapse pressures (1).

Mode thr e, collapse by general instability, has been found to be of primary improtance in the preliminary

design of pressure vessels (14). Solution of elastic general instability collapse was first undertaken by Tokugawa, who was also the first to identify this mode. His formula for collapse pressure, (which was later confirmed by Bryant), is as follows:

$$P_{cr} = \frac{Eh}{R_m}\left[\frac{\lambda^4}{\left(n^2 - 1 + \frac{\lambda^2}{2}\right)(n^2 + d^2)^2}\right](n^2 - 1)\frac{EIe}{R^3 L_f}$$

$$\text{where } \lambda = \frac{\pi R}{L_b}$$

The original Tokugawa formula as given in (19) has a number of extra terms in it which are seen to be negligible for geometries of general interest.

The lengthy formulas developed by Kendrick for elastic general instability are given in (14), however, a graphical solution of Kendrick's equation as developed by Reynolds, and refined by Ball, is included as figures A-2 and A-3. In this solution, the sum of two effects, each represented by a graph, is used to determine collapse pressure. Figure 2 is the shell effect and figure 3 the frame effect which reflects the bending stiffness of the cylinder. Figures 2 and 3 also indicate the Bryant-Tokugawa solution. Collapse pressure by either set of curves is found in the following manner:

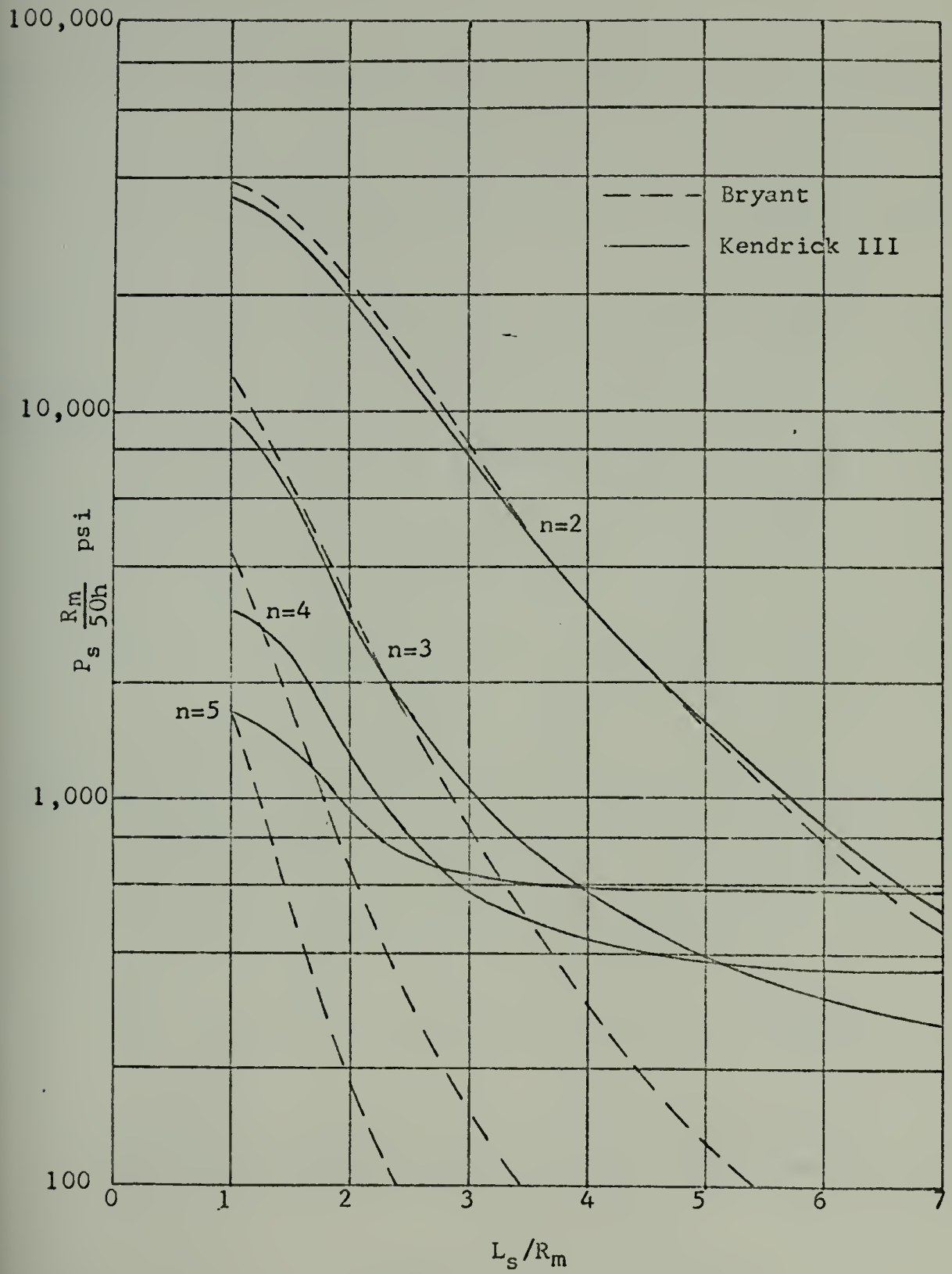


FIGURE A-2 (SHELL EFFECT)

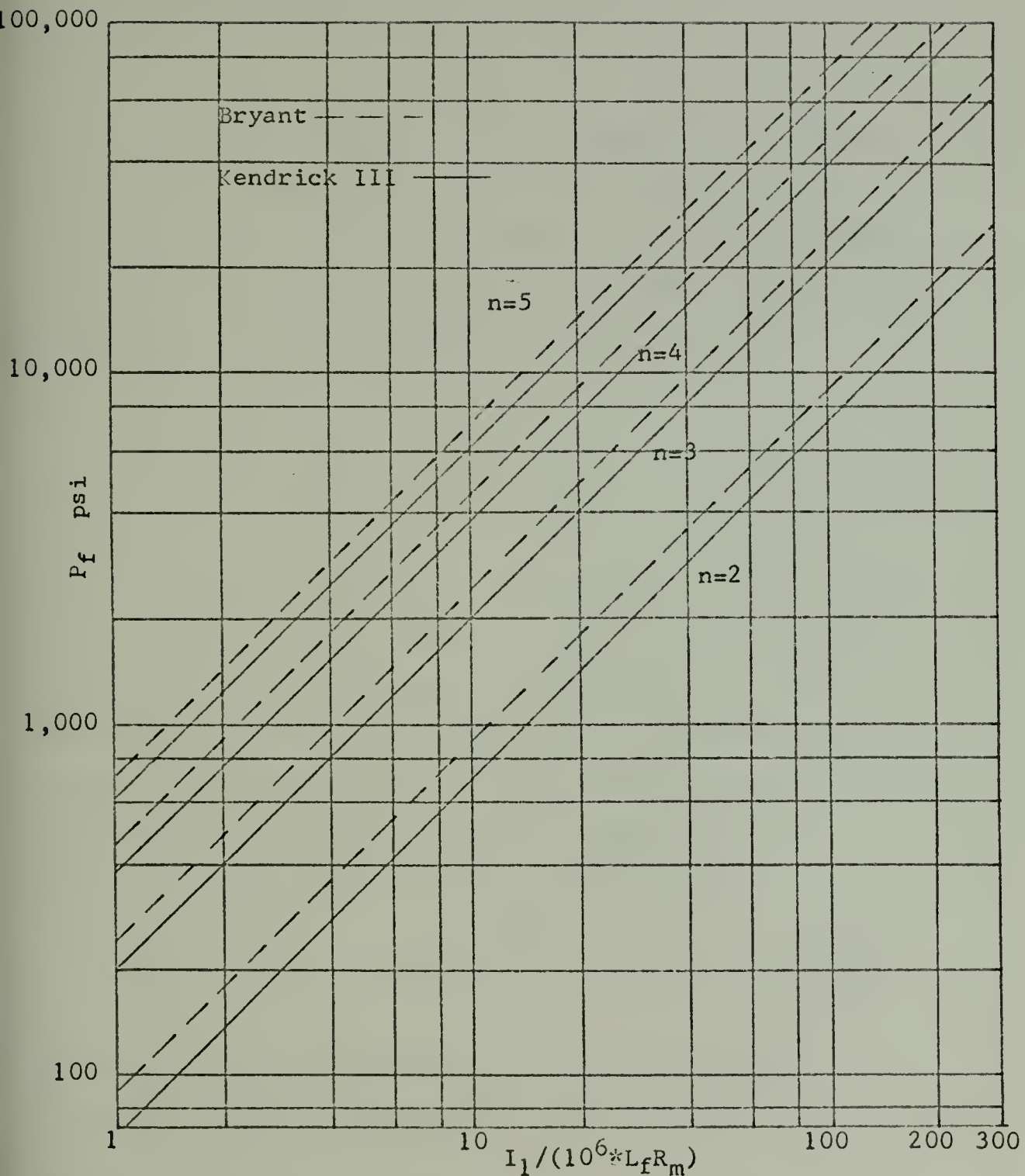


FIGURE A-3 (FRAME EFFECT)

$$p_c = p_s \left(\frac{L_b}{R}, \frac{h}{R}, n \right) + p_f \left(\frac{Ie}{R^3 L_f}, n \right)$$

In order to round out this development of stiffened cylindrical pressure vessel collapse theories, mention must be made of the effect of actual physical manufacturing conditions. In all but the smallest and most painstakingly fabricated cylinders, imperfect production processes are the cause of a number of collapse strength modifiers. Those modifiers introduced into the material itself are in the form of stresses due to fabrication, rolling, and welding of flat plating. In thick homogeneous plating, the homogeneity itself may be questionable and thus cause stress concentrations. A major strength modifier is the imperfect circularity of most welded cylinders. This modifier has received some theoretical consideration as it has a considerable effect on the resistance of a stiffened cylinder to failure by general instability. A formula for this is due in part to Kendrick (2):

$$\sigma_{max} = \frac{p_1 R}{A_b} b \left[\frac{1 + \frac{.85}{B}}{1 + \beta} \right] + \frac{Ec}{R_1^2} (n^2-1) e \left(\frac{p_1}{p_{cr} - p_1} \right)$$

As long as the σ_{max} is below material yield stress, collapse by general instability is not indicated.

This section has been included to give a brief background of the development of theoretical collapse predictors for stiffened cylindrical shells. In particular, a history of the variables of λ and ψ is helpful in understanding their use in this paper.

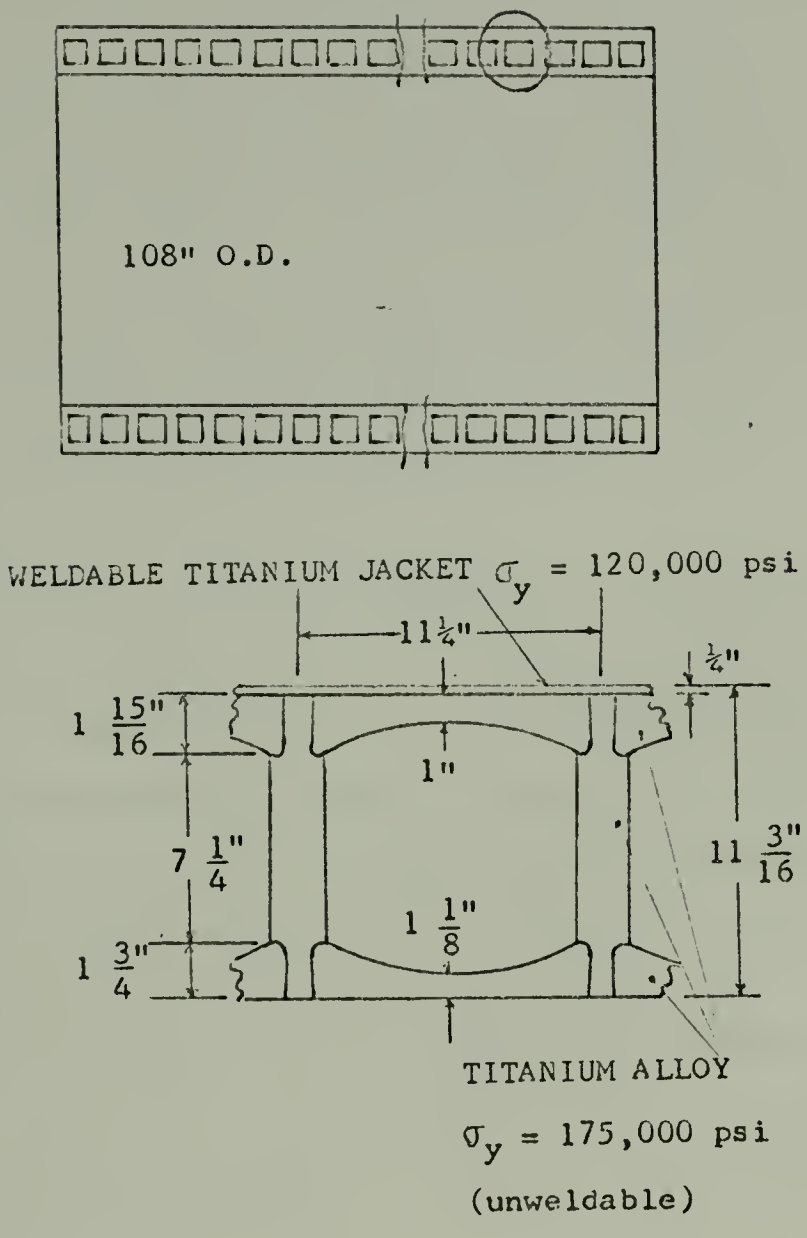
APPENDIX B

DISCUSSION OF VERY LOW LAMBDA DESIGN

As noted in Section IV, overall structural efficiency approaches its highest value as ψ goes to 1.0 and λ goes to 0. For T-shaped frames of the geometry used in this report, λ values in the area below .2 may be considered very low lambda design. For other frame geometries, this may be higher. For example, the sandwich construction in figure A-4 has a λ value of about .337. The frames used there have thicker webs than flanges, whereas the reverse is true for the frames of this report. The membrane shell design, and other complications resulting from the use of high strength titanium also plays a part in deciding the optimum λ value.

It was noted in Section IV that the danger of general instability collapse would be greatly increased by the use of a very low lambda design. This has been shown to be true in model tests of cylindrical shells with geometries similar to that of figure A-4. In fact, in the testing of these models, a new mode of collapse was observed - that of plastic general instability - and new theories had to be developed to predict it. It may also be observed that collapse of these models took place at ψ value in excess of 3.0. This suggests that at very low lambda

values, ψ , becomes a rather meaningless term unless the framing is very small in comparison to the shell thickness. Indeed, the large effect of framing at these λ values throws into doubt the meaningfulness of λ itself. This doubt has been substantiated but not fully explained in reference (6).



SCHEMATIC SKETCH of an OCEANOGRAPHIC VEHICLE WITH . 9-FOOT OUTER DIAMETER from DTMB 1677

FIGURE A-4

APPENDIX C

FRAME PARAMETERS

A T-shaped frame was used in the calculations leading to figures 5-10. The proportions of this frame were taken from a work by McGinley (11) and are as follows:

frame width/frame depth	=	1.0
frame width/web thickness	=	8.5
flange thickness/web thickness	=	1.7

Figure A-5 is a graph of b/D versus λ for the frames for both formulas #92A and Wenk's. Since the geometry of optimized vessels was found to be a function of required collapse pressure,and not required diameter, the optimized frame dimensions should be proportional to the major cylinder dimensions, hence the b/D parameter. Figure A-4 may be used in conjunction with figures 8, 9, and 10 to determine the frame dimensions. Care should be taken, however, to match yield formulas and frame factors when using these figures together.

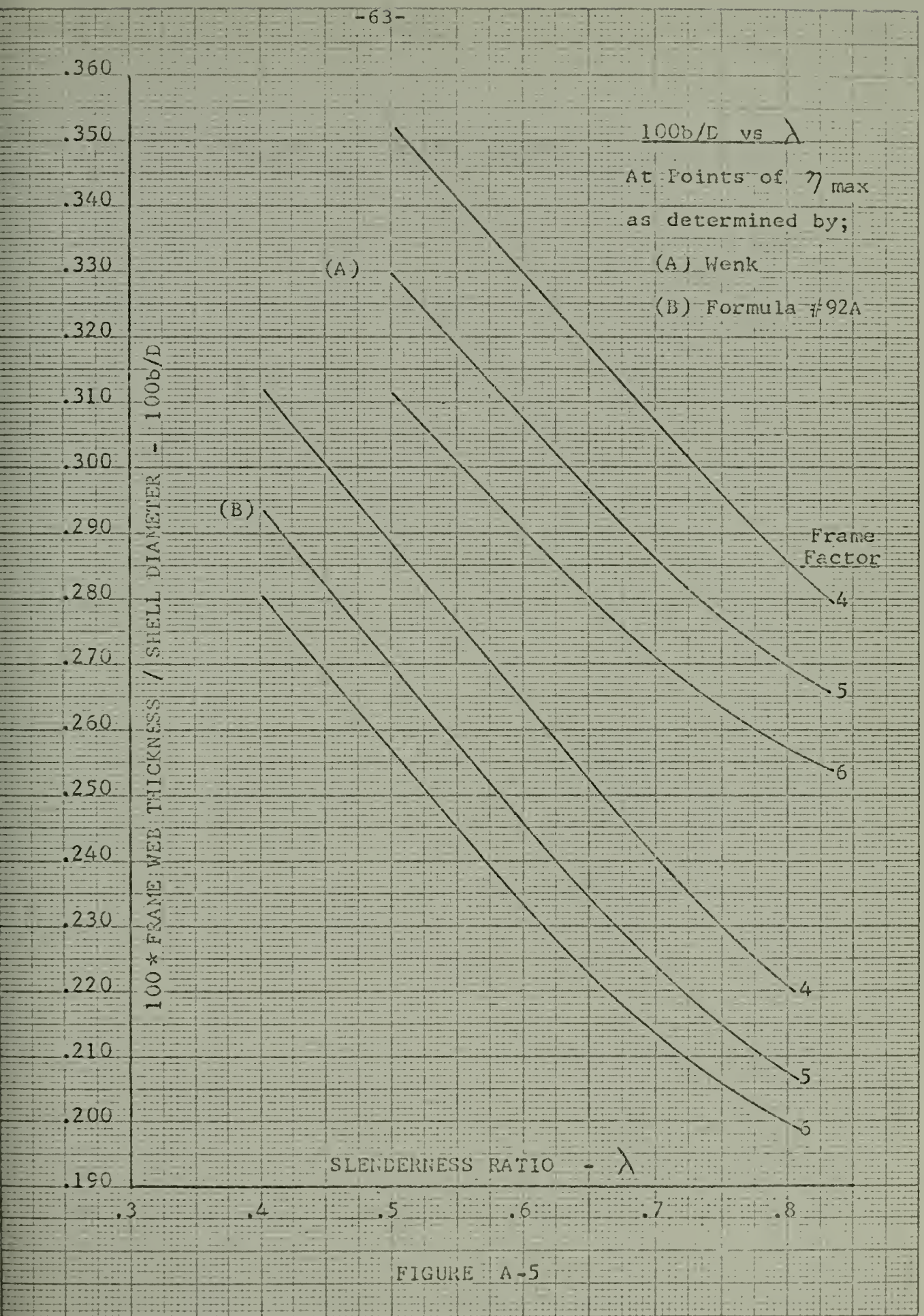

100b/D vs λ
At Points of η max
as determined by;
(A) Wenk
(B) Formula #92A
(A)
(B)
Frame Factor
4
5
6
4
5
6
100 * FRAME WEB THICKNESS / SHELL DIAMETER = 100b/D
.360
.350
.340
.330
.320
.310
.300
.290
.280
.270
.260
.250
.240
.230
.220
.210
.200
.190
SLENDERNESS RATIO = λ
.3
.4
.5
.6
.7
.8

FIGURE A-5

APPENDIX D

EXAMPLES OF DESIGN ITERATION

Example 2:

This example is included to indicate the results of the iterative method when applied to relatively low pressure vessels.

Given: Use of #92A Frame Factor 5

σ_y = 30,000 psi

p_c = 400 psi

D = 40 ft.

Steps:

1. Initial h/D_m guess from hoop stress

(h/D_m) = 400/(2*30,000)

(h/D_m) = .00666

λ = .495

ψ = 1.30

p_c = 1.30*400 = 520 psi which is too high

2. Reduce h/D_m by division

$p_{c1} = \frac{400}{1.3} = 308$ psi

(h/D_m) = .00512

λ = .565

ψ = 1.39

p_c = 1.39*308 = 428 psi which is still too high

$\psi_{act} = \frac{428}{400} = 1.07$

3. Reduce h/D_m again

$$p_{c2} = \frac{308}{1.07} = 288 \text{ psi}$$

$$(h/D_m) = .00480$$

$$\lambda = .584$$

$$\psi = 1.41$$

$$p_c = 1.41*288 = 406 \text{ psi}$$

which is close enough

Answer:

$$\lambda = .584$$

$$h/D_m = .00480$$

$$\eta = 1.27*\frac{30,000}{80,000}$$

$$\eta = .476$$

$$\psi = 1.41$$

Results:

$$D = 40 \text{ ft.}$$

$$h = 2.3 \text{ in.}$$

$$L = 4.5 \text{ ft.}$$

$$\frac{Wt}{\Delta} = \frac{p_c}{3720\eta} = \frac{406}{3720(.476)}$$

$$\frac{Wt}{\Delta} = .23$$

Example 3:

This example is exactly like example 2 except that a higher strength material is used.

Given: Use of #92A Frame Factor 5

σ_y = 50,000 psi

p_c = 400 psi

D = 40 ft.

Answer: λ = .825

h/D_m = .00232

η = $1.11 * \frac{50,000}{80,000}$

η = .694

ψ = 1.735

Results: D = 40 ft.

h = 1.11 in.

L. = 1.83 ft.

$\frac{Wt}{\Delta}$ = .157

Example 4:

This example is included to indicate the use of the frame parameter curves in Appendix C.

Given: Use of #92A Frame Factor 6

σ_y = 100,000 psi

D = 40 ft.

p_c = 1500 psi

Steps:

1. Initial h/D_m guess

(h/D_m) = 1500/(2*100,000)

(h/D_m) = .007

λ = .48

ψ = 1.33

p_c = 1.33*1500 = 2000 psi

2. Reduce h/D_m

p_{c1} = $\frac{1500}{1.33}$ = 1130

(h/D_m) = 1130/(2*100,000)

(h/D_m) = .0056

λ = .54

ψ = 1.40

p_c = 1.40*1130 = 1580 psi

ψ_{act} = $\frac{1580}{1500}$ = 1.05

3. Reduce h/D_m again

$$p_{c2} = \frac{1130}{1.05} = 1075 \text{ psi}$$

$$(h/D_m) = 1075/(2*100,000)$$

$$(h/D_m) = .00537$$

$$\lambda = .55$$

$$\psi = 1.41$$

$$p_c = 1.41*1075 = 1515 \text{ psi}$$

close enough

Answer:

$$\lambda = .55$$

$$(h/D_m) = .00537$$

$$\eta = \frac{100,000}{80,000} = 1.59$$

$$\psi = 1.41$$

$$\frac{b}{D} = .00248$$

Results:

$$D = 40 \text{ ft.}$$

$$h = 2.6 \text{ in.}$$

$$L = 1.42 \text{ ft.}$$

$$\frac{Wt}{\Delta} = .252$$

Frame Dimensions:

$$b = 1.18 \text{ in.}$$

$$w = 8.05 \text{ in.}$$

$$f = 10.1 \text{ in.}$$

$$t_2 = 2.05 \text{ in.}$$

APPENDIX E

FORMULAS AND DERIVATIONS

In this appendix, the formulas used in the computer programs in Appendix F are listed. Also, the derivation of formulas (A) and (B), Section II, and the solution of the Hencky-von Mises criterion for unstiffened tubes are presented.

FRAME FORMULAS: The formulas below are used to predict frame failure by instability. Collapse of the frame by yielding was not investigated as the frames were not required to hold up after shell failure and the maximum stress in the frame is always less than the maximum stress in the shell (19). However, this stress relationship may not hold in the very low lambda region.

Foppl Formula:

$$p_c = \frac{24EI_b}{D_b{}^3 L_1}$$

Formula #88
von Sanden and Gunther:

$$p_c = \frac{24EI_b\ (1+\beta)}{D_b{}^3 b\ \left(1 + \frac{.85\beta}{B}\right)}$$

$$\text{where } B = \frac{bh}{A + bh}$$

$$\theta = 1.82 \frac{L/D_m}{h/D_m}$$

$$\beta = 2\frac{L}{b} \frac{NB}{\theta}$$

$$N = \frac{\cosh\theta - \sin\theta}{\sinh\theta + \sin\theta}$$

SHELL FORMULAS: The first formula below predicts collapse of the shell between frames due to instability. The next four predict collapse due to yielding of the material according to different yield criteria.

Formula #92
von Sanden and Gunther

Failure due to axial stress exceeding yield stress:

$$p_c = 2\frac{h}{D_m}\sigma_y \left(\frac{1}{.5 + 1.81KC} \right)$$

$$K = \frac{\sinh\theta - \sin\theta}{\sinh\theta + \sin\theta}$$

$$C = \frac{.85 - B}{1 + \beta}$$

B and β are defined in Formula #88

Formula #92A
von Sanden and Gunther

Failure due to circumferential stress exceeding yield stress:

$$p_c = 2\frac{h}{D_m}\sigma_y \left(\frac{1}{1 + HC} \right)$$

$$H = -2\,\frac{(1+M)\,\sinh\frac{\theta}{2}\cos\frac{\theta}{2} + (1-M)\,\cosh\frac{\theta}{2}\sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

$$M = \frac{3\mu^2}{1-\mu^2}$$

C is defined in Formula #92

Wenk's Formula

This formula was actually developed by Wenk and others at DTMB using #92 and #92A to determine the principle stresses. These were applied at mid-bay and mid-thickness using the Hencky-von Mises yield criterion.

$$p_c = 2\frac{h}{D_m}\sigma_y\,\frac{1}{\sqrt{F^2 - .5F + .25}}$$

$$F = 1 + 2QC$$

$$Q = -\,\frac{\sinh\frac{\theta}{2}\cos\frac{\theta}{2} + \cosh\frac{\theta}{2}\sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

θ and C are defined in Formula #92

Lunchick's Formula

This formula uses the principle stress relations developed by Salerno and Pulos rather than #92 and #92A (9). It applies them to the mid-bay location using the Hencky-von Mises criterion, however, failure is not predicted until the yielding which occurs first in the outer surface develops into a fully plastic hinge. Lunchick's formula predicts this yield pressure, but curves (included here

as figure A-6) must be used to determine the actual collapse pressure.

$$p_c = 2\frac{h}{D_m}\sigma_y \sqrt{\frac{1}{X^2 - XZ + Z^2}}$$

$$X = 1 + HC$$

$$Z = .5 + 3.62TC$$

$$T \cong - \frac{\sinh\frac{\theta}{2}\cos\frac{\theta}{2} - \cosh\frac{\theta}{2}\sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

θ, C, and H are defined in Formula #92A

Entry into the curves requires computation of the following ratios:

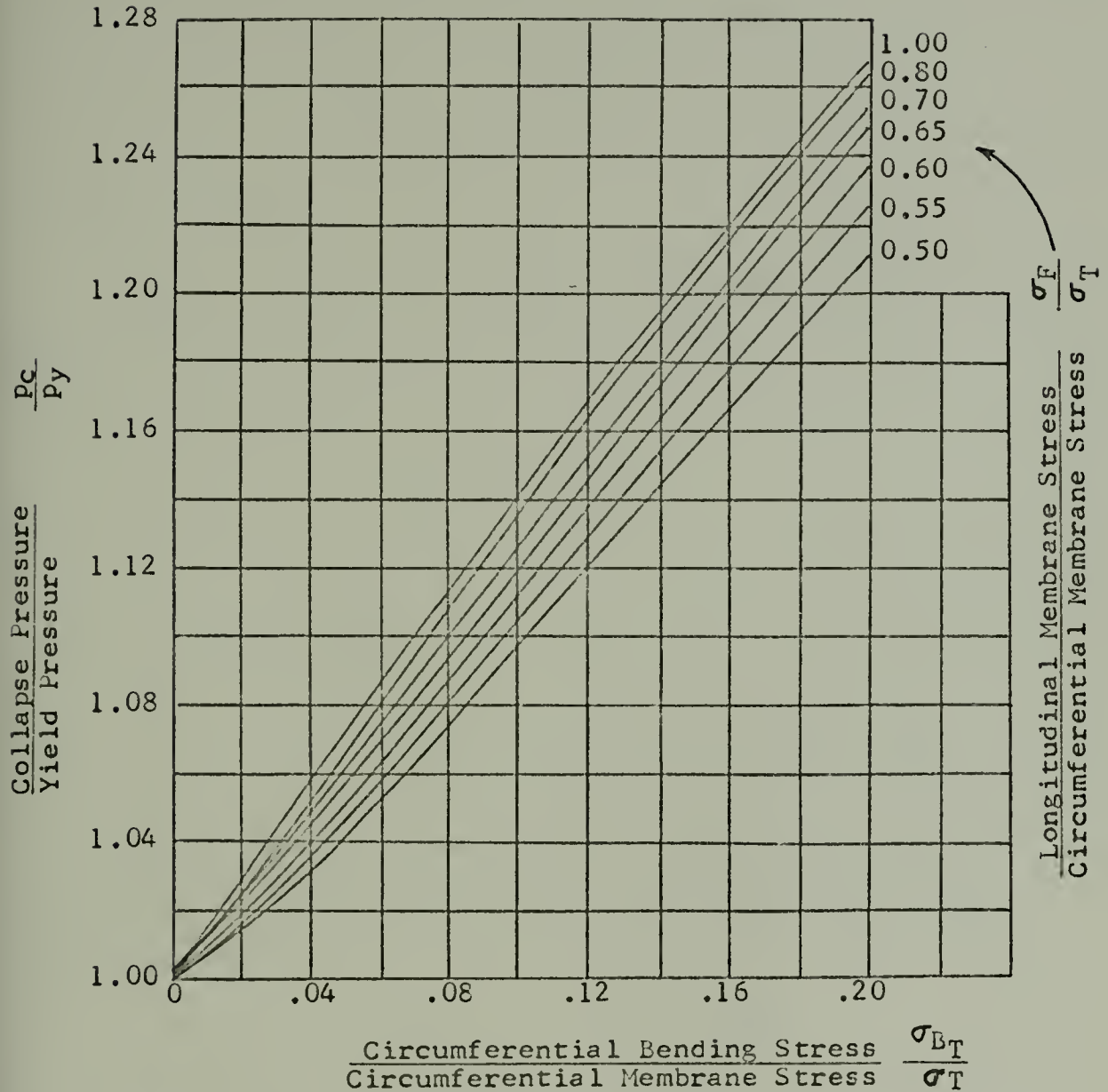
$$\frac{\text{Circumferential Bending Stress}}{\text{Circumferential Membrane Stress}} = \frac{\sigma_{BT}}{\sigma_T}$$

$$\frac{\text{Longitudinal Membrane Stress}}{\text{Circumferential Membrane Stress}} = \frac{\sigma_F}{\sigma_T}$$

$$\sigma_F = \frac{D_m\, p_c}{4h}$$

$$\sigma_T = 2\sigma_F(1 + 2QC)$$

$$\sigma_{BT} = 2\sigma_F\, TC$$



Curves for Lunchick's Collapse Pressure Factor - From DTMB Report 1291

FIGURE A-6

GENERAL INSTABILITY FORMULAS: While Tokugawa's formula and graphs of Kendrick's third solution for general instability have been given in Appendix A, Bryant's simplification of Kendrick's first solution has been used in the computer program.

Bryant's Formula:

$$p_{cr} = 2\frac{h}{D_m}E \left(\frac{\lambda^4}{(n^2 - 1 + \frac{\lambda^2}{2})(n^2 + \lambda^2)} \right)$$

$$+ \frac{n^2 - 1}{R_m^3 L_1} EI_1$$

$$\lambda = \frac{\pi R}{L_b} \quad \text{an integer}$$

n = the integer which gives the lowest p_{cr}.

Since out of roundness effects general instability collapse, a formula taken from (2) has been used in the following form:

$$\sigma_{max} = \frac{p_c R}{A_b} b \left[\frac{1 + \frac{.85\beta}{B}}{1 + \beta} \right]$$

$$+ \frac{Ec}{R_1^2}(n^2 - 1)e \left(\frac{p_c}{p_{cr} - p_c} \right)$$

e = eccentricity

p_c= desired collapse pressure

n, p_{cr}, β, and B are determined in previous formulas.

λ_m FROM PRINCIPLE STRESS CRITERION AT MID-BAY EXTERIOR, FORMULA #92A

1. Formula #92A

$$p_c = \frac{2\frac{h}{D_m}\sigma_y}{1 + H\left(\frac{.85 - B}{1 + \beta}\right)}$$

2. $$\psi = \frac{p_c}{2\frac{h}{D_m}\sigma_y}$$

$$\psi = 1 \text{ @ } \frac{1}{1 + H\left(\frac{.85 - B}{1 + \beta}\right)} \doteq 1$$

$$1 + H\left(\frac{.85 - B}{1 + \beta}\right) = 1$$

$$H\left(\frac{.85 - B}{1 + \beta}\right) = 0$$

$$H = 0$$

$$H \cong - \frac{3 \sinh\frac{\theta}{2} \cos\frac{\theta}{2} + \cosh\frac{\theta}{2} \sin\frac{\theta}{2}}{\sinh + \sin}$$

$$H = 0 \text{ @ } \theta = 3.74 \text{ radians}$$

$$\theta = (1.82\ L/D_m)/\sqrt{h/D_m}$$

$$L/D_m = 2.055\sqrt{h/D_m}$$

$$h/D_m = .236(L/D_m)^2$$

3. $$\lambda = \sqrt[4]{\frac{(L/D_m)^2}{(h/D_m)^3}} \sqrt{\frac{\sigma_y}{E}}$$

$$\lambda_m = \sqrt{\frac{2.055\ D_m}{h}} \frac{\sigma_y}{E}$$

Equation (A), Section II

λ_m FROM HENCKY-VON MISES CRITERION AT MID-BAY AND MID-THICKNESS,
VON SANDEN AND GUNTHER FORMULAS

1. Wenk's Formula

$$p_c = \frac{2h\sigma_y}{D_m} \quad \frac{1}{\sqrt{F^2 - \frac{F}{2} + \frac{1}{4}}}$$

$$F = 1 + 2Q\left(\frac{.85 - B}{1 + \beta}\right)$$

$$Q = -\frac{\sinh\frac{\theta}{2}\cos\frac{\theta}{2} + \cosh\frac{\theta}{2}\sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

2. $\psi = \dfrac{p_c}{2\frac{h}{D_m}\sigma_y}$

$$\psi = 1.154 @ \frac{1}{\sqrt{F^2 - \frac{F}{2} + \frac{1}{4}}} = 1.154$$

$$F^2 - \frac{F}{2} + \frac{1}{4} = .75$$

$$F = .25 \pm .75$$

$$\therefore 1 = 1 + 2Q\left(\frac{.85 - B}{1 + \beta}\right)$$

$$\therefore Q = 0 @ \theta = 4.68 \text{ radians}$$

$$\theta = (1.82\ L/D_m) / \sqrt{h/D_m}$$

$$L/D_m = 2.565\sqrt{h/D_m}$$

$$h/D_m = .152\ (L/D_m)^2$$

3. $$\lambda = \sqrt[4]{\frac{(L/D_m)^2}{(h/D_m)^3}} \sqrt{\frac{\sigma_y}{E}}$$

$$\lambda_m = \sqrt{\frac{2.565\ D_m}{h} \frac{\sigma_y}{E}}$$

Equation (B), Section II

SHELLS [illegible] WIDELY SPACED STIFFENERS

1. He[illegible]-von Mises Criterion

σ_1, [illegible], σ_3 - three principle stresses

$$2\sigma_{[illegible]} = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

2. Fo[illegible] unstiffened tube:

$$\sigma_1 = 2\sigma_2 \qquad \sigma_3 = 0$$

$$2\sigma_y^2 = \sigma_2^2 + \sigma_2^2 + 4\sigma_2^2$$

$$\sigma_y^2 = 3\sigma_2^2$$

$$\sigma_2 = \frac{1}{1.732\ \sigma_y} = .577\sigma_y$$

$$\sigma_1 = \frac{2}{1.732\ \sigma_y} = 1.154\sigma_y$$

APPENDIX F

COMPUTER PROGRAMS

Two computer programs were used to produce theoretical data for this report. Program 1 utilizes equation (A) of Section II to determine curves of h/D_m versus λ_m for eight values of σ_y. Since equation (A) is based on the θ value at which the variable H in formula #92A goes to zero, the program first solves for that θ and then goes on to vary h/D_m and σ_y, solving for λ_m. This program was used only to define the upper limits of the yield failure area and so, is not actually necessary for the iterative optimization proceedure of Section IV. A flow chart and variable chart are included with the program listing and printout.

Program 2 follows a scatter-gun philosophy in that it determines collapse pressures of given geometries for all three modes of failure by various formulas. All of the formulas are mentioned in Appendix A and completely defined in Appendix E. The input of this program consists of outside diameter, desired range of collapse pressures, frame proportions, material properties, and range of values. Shell thickness is determined by the hoop stress relation and frame size by a modified Foppl formula. Program 2 was not intended to be used to produce a complete

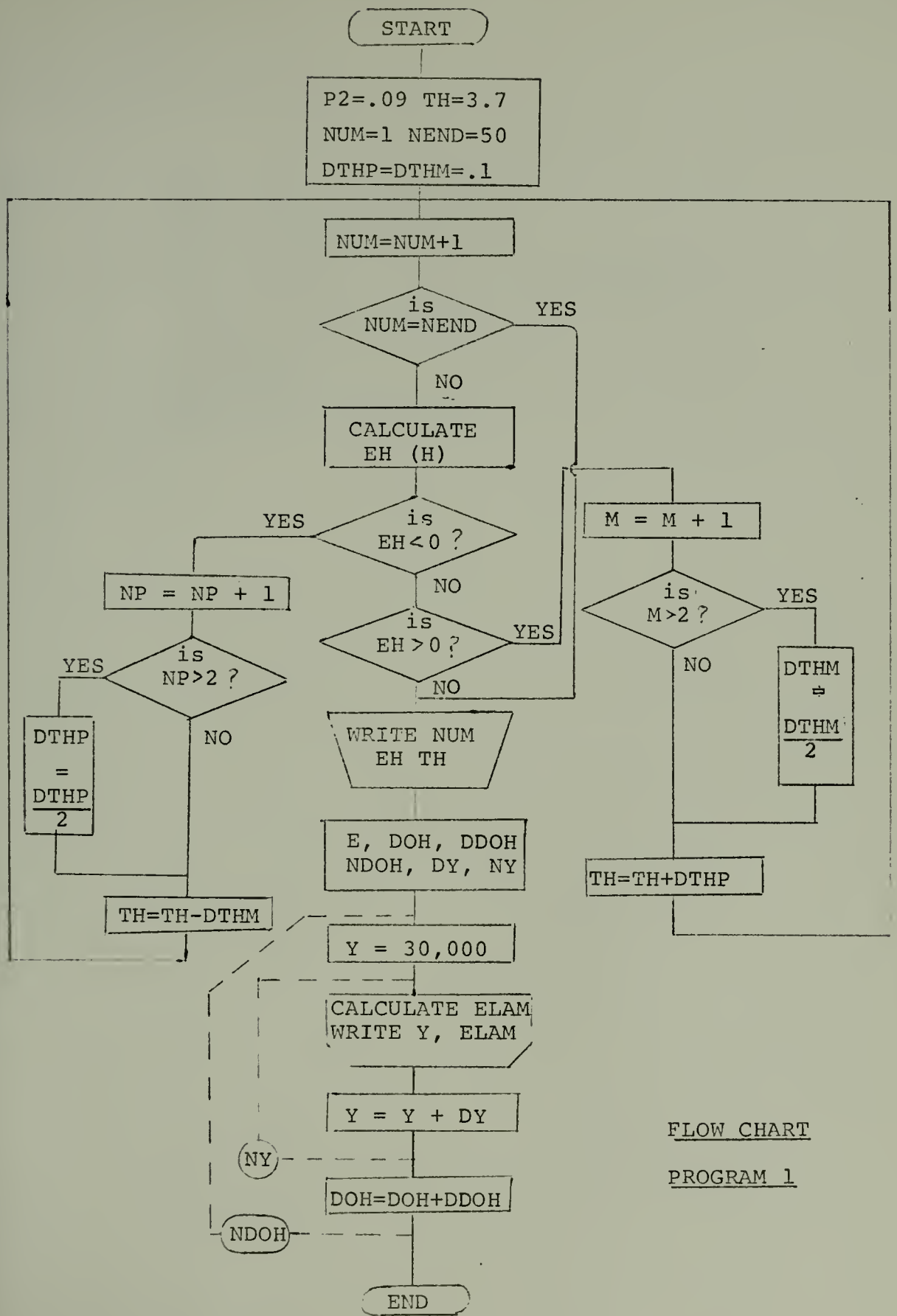
optimized design for a given set of requirements. Its main use, in solving for and printing collapse pressures for all modes, is pointing up trends and areas where various formulas fail. It has provisions for any number of solutions to be printed and any geometry to be investigated. In short, it is meant to be used as a tool by a designer who optimizes, using judgment and experience as well as computer time. A flow chart and variable chart are included with the program listing and a sample of its printout.

PROGRAM 1

```
C     THIS FROGRAM  DETERMINES LAMBDA MAX FOR CHANGING H/D
C     AND YIELD STRESS USING A FORMULA DERIVED FROM 92A.
C
C
      P2 = .09
      NUM = 1
      NEND = 50
      TH = 3.7
      DTHP = .1
      DTHM = .1
    1 NUM = NUM + 1
      IF(NUM.EQ.NEND) GO TO 30
      THT = TH/2.
      DEN = SINH(TH)+SIN(TH)
      EH0 = (1.+(3*P2/(1-P2))**.5)*SINH(THT)*COS(THT).
      EH1 = (1.-(3*P2/(1-P2))**.5)*COSH(THT)*SIN(THT)
      EH = -2*(EH0+EH1)/DEN
      IF(EH) 19,30,9
    9 NP = NP+1
      IF(NP.EQ.3) GO TO 10
      DTHP = DTHP/2
   10 TH = TH-DTHM
      M = 1
      GO TO 1
   19 M = M+1
      IF(M.EQ.3) GO TO 20
      DTHM = DTHM/2
   20 TH = TH + DTHP
      NP = 1
      GO  TO 1
   30 WRITE(6,180) NUM,EH,TH
  180 FORMAT(' AFTER',I3,' ITERATIONS, H IS',F8.6,' AND THETA
      E = 30000000.0                                    IS',F6.3)
      DOH = 0.0
      DDOH = 40.0
      NDOH = 15
      DY = 10000.0
      NY = 8
      DO 50 N = 1,NDOH
      WRITE(6,205) DOH
      Y = 30000.0
      DO 40 M = 1,NY
      ELAM = (TH/1.82*DOH*Y/E)**.5
      WRITE(6,210) Y,ELAM
   40 Y = Y+DY
   50 DOH = DOH + DDOH
  205 FORMAT(' LAMBDAS FOR DM/H = ',F7.2,' ARE ')
  210 FORMAT(2F12.4)
      END
```

PRINTOUT OF PROGRAM 1 (Sample)

```
AFTER 50 ITERATIONS, H IS-.000000 AND THETA IS 3.74
LAMBDAS FOR DM/H =     0.0  ARE
 30000.0000          0.0
 40000.0000          0.0
 50000.0000          0.0
 60000.0000          0.0
 70000.0000          0.0
 80000.0000          0.0
 90000.0000          0.0
100000.0000          0.0
LAMBDAS FOR DM/H =   40.00 ARE
 30000.0000          0.2867
 40000.0000          0.3311
 50000.0000          0.3702
 60000.0000          0.4055
 70000.0000          0.4380
 80000.0000          0.4683
 90000.0000          0.4967
100000.0000          0.5235
LAMBDAS FOR DM/H =   80.00 ARE
 30000.0000          0.4055
 40000.0000          0.4683
 50000.0000          0.5235
 60000.0000          0.5735
 70000.0000          0.6194
 80000.0000          0.6622
 90000.0000          0.7024
100000.0000          0.7404
LAMBDAS FOR DM/H =  120.00 ARE
 30000.0000          0.4967
 40000.0000          0.5735
 50000.0000          0.6412
 60000.0000          0.7024
 70000.0000          0.7587
 80000.0000          0.8110
 90000.0000          0.8602
100000.0000          0.9068
LAMBDAS FOR DM/H =  160.00 ARE
 30000.0000          0.5735
 40000.0000          0.6622
 50000.0000          0.7404
 60000.0000          0.8110
 70000.0000          0.8760
 80000.0000          0.9365
 90000.0000          0.9933
100000.0000          1.0470
```



FLOW CHART

PROGRAM 1

NOMENCLATURE OF PROGRAM 1

DOH	-	D_m/h
DDOH	-	Incremental change in D_m/h
NDOH	-	Number of changes in D_m/h
E	-	Young's Modulus
EH	-	Variable H of yield failure formulas
ELAM	-	λ_m
P2	-	Poisson's Ratio squared
TH	-	Variable θ of yield failure formulas
Y	-	Material σ_y
DY	-	Incremental change in σ_y
NY	-	Number of changes in σ_y

PROGRAM 2

```
C     THIS PROGRAM DETERMINES THEORETICAL COLLAPSE PRESSURES
C     FOR STIFFENED CYLINDRICAL PRESSURE VESSELS. THREE MODES
C     OF COLLAPSE ARE INVESTIGATED WITH EMPHASIS ON YIELD
C     PRECIPITATED BUCKLING BETWEEN THE FRAMES. FRAME SIZE
C     IS DETERMINED USING A MODIFIED FOPPL FORMULA. ALL INPUTS
C     ARE HANDLED BY MAIN PROGRAM CARDS - NO DATA CARDS ARE
C     USED.

      MF = 6
      D = 480.0
      PC = 300.0
      DO 700 NU = 1,14
      Y = 80000.0
      E = 30000000.0
      R1 = 1.0
      R2 = .11764
      R3 = 1.7
      P = .3
      P2 = P**2
      SR = .38
      IF(PC.LT.1000.0) SR = .46
      DSR = .02
      NSR = 20
      T = D*PC/(2*Y)
      DM = D-T
      TOD = T/DM
      WRITE(6,160) T,TOD
      DO 500 IN = 1,NSR
      EL = SR**2*DM*E/Y*(T/DM)**1.5
      FRAMD1 = 12.0
      DFRP = 1.0
      DFRM = 1.0
      NUM = 1
      NEND = 70
      RATIO = EL*PC/(24*E*MF)
    1 NUM = NUM + 1
      IF(NUM.EQ.NEND) GO TO 30
      F = FRAMD1*R1
      B = FRAMD1*R2
      T2 = B*R3
      W = FRAMD1 - T2
      YB = YENTRD(B,T,B,W,T2,F)
      YBI = YMOM(B,T,B,W,T2,F,YB)
      DBT = (D-2*(YB+T))**3
      RAT = YBI/DBT
      IF(RAT-RATIO) 19,30,9
    9 NP = NP+1
```

```
      IF(NP.EQ.3) GO TO 10
      DFRP = DFRP/2
   10 FRAMD1 = FRAMD1 - DFRM
      M = 1
      GO TO 1
   19 M = M+1
      IF(M.EQ.3) GO TO 20
      DFRM = DFRM/2
   20 FRAMD1 = FRAMD1 + DFRP
      NP = 1
      GO TO 1
   30 YF = YENTRD(0.0,0.0,B,W,T2,F)
      Y1 = YENTRD(EL,T,B,W,T2,F)
      YI = YMOM(0.0,0.0,B,W,T2,F,YF)
      Y1I = YMOM(EL,T,B,W,T2,F,Y1)
      YBIOYI = YBI/YI
      Y1IOYI = Y1I/YI
      RONE = (D-2*(T+Y1))/2
      DB = D-2*(T+YB)
      A = F*T2 + B*W
      AB = A+B*T
      EB = B*T/AB
      THETA = 10*(12-12*P2)**.25
      TH = THETA*EL/DM/(100*T/DM)**.5
      THT = TH/2
      DEN = SINH(TH)+SIN(TH)
      EN = (COSH(TH)-COS(TH))/DEN
      EK = (SINH(TH)-SIN(TH))/DEN
      BETA = 2.*EL*EN*EB/(B*TH)
      EH0 = (1+(3*P2/(1-P2))**.5)*SINH(THT)*COS(THT)
      EH1 = (1-(3*P2/(1-P2))**.5)*COSH(THT)*SIN(THT)
      EH = -2*(EH0+EH1)/DEN
      EQ = -(SINH(THT)*COS(THT)+COSH(THT)*SIN(THT))/DEN
      ET = -(SINH(THT)*COS(THT)-COSH(THT)*SIN(THT))/DEN
      SUBDEN = (.85-EB)/(1+BETA)
      EF = 1. + 2*EQ*SUBDEN
      EX = 1. +EH*SUBDEN
      EZ = .5+3.62*ET*SUBDEN
      PTOK = 24*E*Y1I/(DM**3*EL*(1-P2))
      PWIND = 2.6*E*(T/DM)**2.5/(EL/DM-.45*(T/DM)**.5)
      DENONE = (1+(.85*BETA/EB))/(1+BETA)
      DENUM = 2*T*Y/DM
      PY88 = 24*E*YBI/(DB**3*B*DENONE)
      SI92 = 1/(.5+1.81*EK*SUBDEN)
      SI92A = 1./(1+EH*SUBDEN)
      SIWENK = 1./(EF**2-EF/2+.25)**.5
      PLUNCH = 2*T*Y/D/(EX**2-EX*EZ+EZ**2)**.5
      PBRY = 2*E*T/DM/850. + 64*E*Y1I/(DM**3*EL)
      SIGOOR = PC*D*B*DENONE/(AB*2)
      SIGOOR = SIGOOR + E*((D-DB)/2.)*4*(PC/(PBRY-PC))/RONE**2
      SIGF = 1000*DM/(4*T)
```

```
      SIGT = SIGF*2*(1.+2*EQ*SUBDEN)
      SIGBT = SIGF*2*ET*SUBDEN
      SIGFOT = SIGF/SIGT
      SIGBOT = SIGBT/SIGT
      R = D/2.
      PI = 3.14159
      VT = PI*(EL*(R**2-(R-T)**2+B*((R-T)**2-(R-T-W)**2)
     1  +F*((R-T-W)**2-(R-T-W-T2)**2))
      WT = VT*.284*12/EL
      VOL = PI*(R/12.)**2*12
      SF92A = SI92A*DENUM*VOL/(WT*100000)
      SFWENK = SIWENK*DENUM*VOL/(WT*100000)
      Write statements for whatever solutions are required
500   SR = SR+DSR
700   PC = PC+100.0
      Format statements
      END
```

```
      FUNCTION SUBROUTINES
```

```
      FUNCTION YENTRD(EL,T,B,W,T2,F)
902   TOP = 2*F*T2*W + F*T2**2 + B*W**2 - EL*T**2
903   BOT = 2*(F*T2 +B*W + EL*T)
904   YENTRD = TOP/BOT
      RETURN
      END
```

```
      FUNCTION YMOM(EL,TLBLWLT2,F,Y)
922   FIRST = (EL*T**3 + B*W**3 + F*T2**3)/12
923   SECOND = EL*T*(T/2+Y)**2 + B*W*(Y-W/2)**2
924   THIRD = F*T2*(T2/2+W-Y)**2
925   YMOM = FIRST + SECOND + THIRD
      RETURN
      END
```

PRINTOUT OF PROGRAM 2 (Sample, continued on next page)

```
T IS  3.0INCHES AND T/DM IS 0.00629
LAMBDA,EL,FRAME FACTOR     0.380   12.883     4
SECOND MOMENTS:F,B,1   200.436   439.394   1481.279
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
28.2750 -28193.383   590.321  1.857     1.488      1.665

LAMBDA,EL,FRAME FACTOR     0.400   14.275     4
SECOND MOMENTS:F,B,1   225.964   485.456   1680.225
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
30.022  -42473.102   609.692  1.830     1.469      1.651

LAMBDA,EL,FRAME FACTOR     0.420   15.738     4
SECOND MOMENTS:F,B,1   253.112   533.708   1893.137
 F ARAE    WINDENBG   FORM88   FORM92    FORM92A    WENK
31.774  -90844.625   628.875  1.801     1.451      1.637

LAMBDA,EL,FRAME FACTOR     0.440   17.273     4
SECOND MOMENTS:F,B,1   281.881   584.134   2120.124
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
33.531   467240.063   647.914  1.770     1.434      1.624

LAMBDA,EL,FRAME FACTOR     0.460   18.879     4
SECOND MOMENTS:F,B,1   312.279   636.729   2361.309
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
35.293   62893.883   666.852  1.736     1.420      1.612

LAMBDA,EL,FRAME FACTOR     0.480   20.556     4
SECOND MOMENTS:F,B,1   344.308   691.478   2616.743
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
37.058   33034.949   685.733  1.700     1.400      1.599

LAMBDA,EL,FRAME FACTOR     0.500   22.305     4
SECOND MOMENTS:F,B,1   377.968   748.369   2886.512
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
38.828   22097.688   704.602  1.662     1.383      1.587

LAMBDA,EL,FRAME FACTOR     0.520   24.125     4
SECOND MOMENTS:F,B,1   413.264   807.395   3170.714
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
40.600   16434.441   723.510  1.622     1.367      1.575

LAMBDA,EL,FRAME FACTOR     0.540   26.016     4
SECOND MOMENTS:F,B,1   450.198   868.543   3468.377
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
42.376   12977.973   742.510  1.581     1.350      1.563

LAMBDA,EL,FRAME FACTOR     0.560   27.979     4
SECOND MOMENTS,F,B,1   488.769   931.803   3782.55
 F AREA    WINDENBG   FORM88   FORM92    FORM92A    WENK
44.154   10653.945   761.666  1.538     1.333      1.551
```

FLANGE,WEB:LENGTH,THICKNESS 9.80 1.96 7.84 1.15
LUNCHICK RATIOS:F/T,B/T .750 .0080

LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.629	2478.0	54958.6	26118.07	1.246	1.327

FLANGE,WEB:LENGTH,THICKNESS 10.10 2.02 8.08 1.19
LUNCHICK RATIOS,F/T,B/T .742 .0095

LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.610	2526.2	55407.4	25657.38	1.253	1.339

FLANGE,WEB:LENGTH,THICKNESS 10.39 2.08 8.31 1.22
LUNCHICK RATIOS:F/T,B/T .735 .0112

LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.592	2571.9	55852.6	25234.33	1.257	1.348

FLANGE,WEB:LENGTH,THICKNESS 10.68 2.13 8.54 1.26
LUNCHICK RATIOS:F/T,B/T .727 .0131

LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.574	2615.4	56292.1	24844.41	1.265	1.356

FLANGE,WEB:LENGTH,THICKNESS 10.95 2.19 8.76 1.29
LUNCHICK RATIOS:F/T,B/T .720 .0151

LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.556	2656.7	56722.0	24483.86	1.267	1.363

FLANGE,WEB:LENGTH,THICKNESS 11.22 2.25 8.98 1.32
LUNCHICK RATIOS:F/T,B/T .713 .0174

LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.538	2695.9	57140.1	24149.51	1.268	1.369

FLANGE,WEB:LENGTH,THICKNESS 11.49 2.30 9.19 1.35
LUNCHICK RATIOS:F/T,B/T .706 .0198

LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.519	2733.4	57544.6	23838.47	1.269	1.374

FLANGE,WEB:LENGTH,THICKNESS 11.75 2.35 9.40 1.38
LUNCHICK RATIOS:F/T,B/T .700 .0225

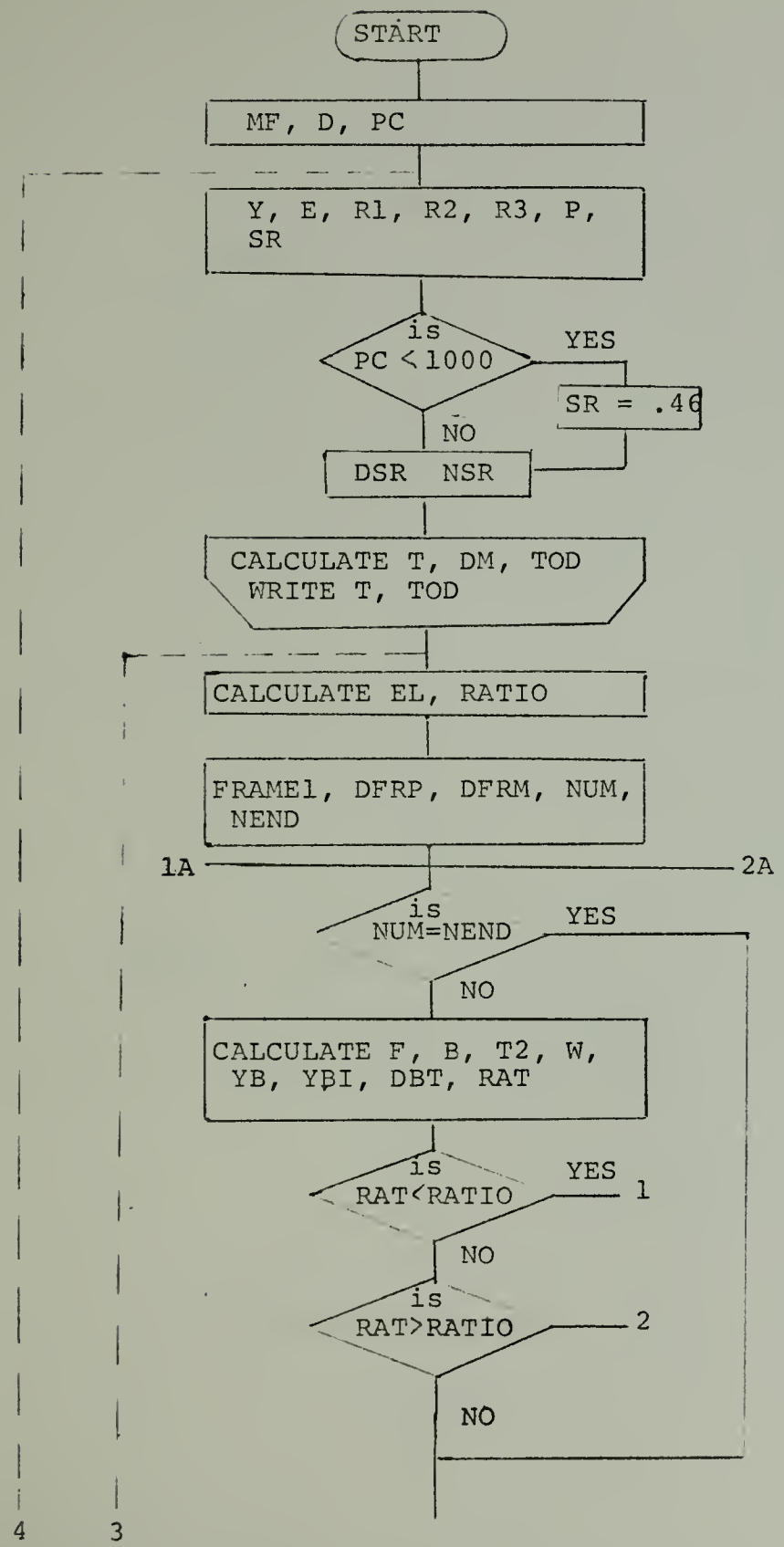
LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.501	2769.0	57932.6	23548.48	1.269	1.378

FLANGE,WEB:LENGTH,THICKNESS 12.00 2.40 9.60 1.41
LUNCHICK RATIOS:F/T,B/T .694 .0253

LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.482	2803.1	58301.6	23277.38	1.267	1.379

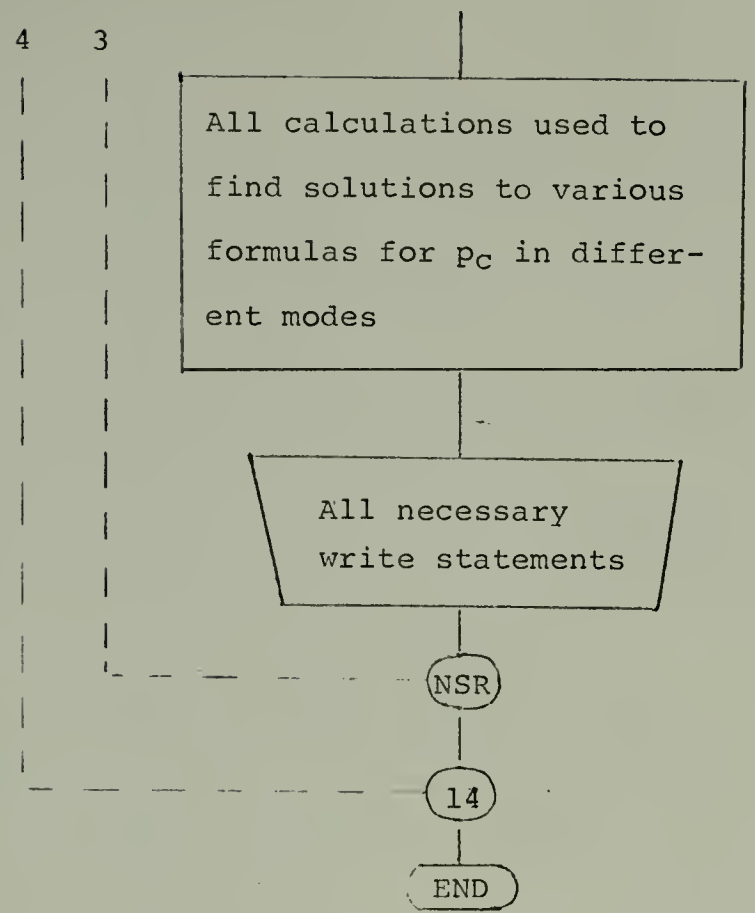
FLANGE,WEB:LENGTH,THICKNESS 12.25 2.45 9.80 1.44
LUNCHICK RATIOS:F/T,B/T .687 .0284

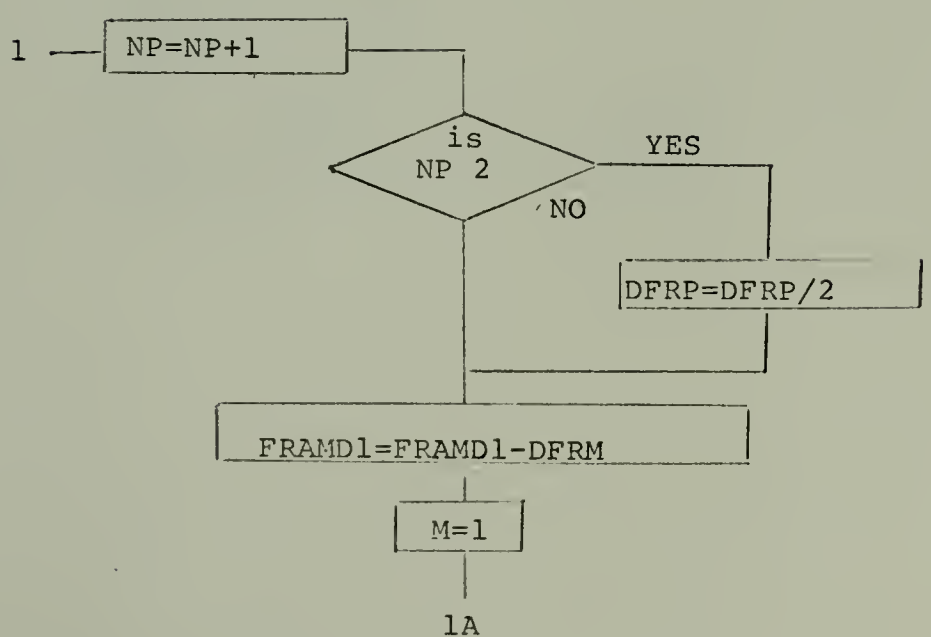
LUNCHICK	BRYANT	OUT ROUND	WT/FT	EF92	EFWENK
1.462	2835.6	58649.1	23023.39	1.265	1.381



FLOW CHART

PROGRAM 2





Loop 2 to 2A is the same as 1 to 1A in its method

NOMENCLATURE OF PROGRAM 2

B	-	Width of frame in contact with shell
D	-	Outside diameter of shell
DSR	-	Incremental change in slenderness ratio
E	-	Young's Modulus
EL	-	Length of shell between frames
F	-	Frame overall width
FRAMD1	-	Frame overall depth
MF	-	Frame Factor
NSR	-	Number of increments of slenderness ratio
P	-	Poisson's Ratio
PBRY	-	Collapse pressure due to Bryant's Formula
PC	-	Required collapse pressure
PLUNCH	-	Collapse pressure due to Lunchick
PTOK	-	Collapse pressure due to Tokugawa
PWIND	-	Collapse pressure due to Windenburg
PY88	-	Collapse pressure due to #88
R1	-	Frame width/Frame depth
R2	-	Web thickness/Frame Depth
R3	-	Flange thickness/Web thickness
SF92A	-	Efficiency due to #92A
SFWENK	-	Efficiency due to Wenk's Formula
SIGFOT	-	σ_F/σ_T for Lunchick Formula
SIGBOT	-	σ_{BT}/σ_T for Lunchick Formula
SI92	-	ψ due to #92
SI92A	-	ψ due to #92A
SIWENK	-	ψ due to Wenk
SR	-	Slenderness Ratio
T	-	Thickness of shell
T2	-	Thickness of flange
W	-	Width of web
WT	-	Weight of vessel per foot length
VOL	-	Volume displaced per foot length
YENTRD	-	Subprogram to find neutral axis
YMOM	-	Subprogram to find I

APPENDIX G

COMPUTED DATA

and

EXPERIMENTAL DATA FROM THE LITERATURE

TABLES OF MAXIMUM VALUES FROM FORMULA #92A AND MAXIMUM PRINCIPLE STRESS CRITERION

	$100h/D_m$	η max/1.2	λ	ψ
Frame Factor 4	.251	.9086	.80	1.785
	.313	.9490	.72	1.664
	.376	.9806	.66	1.574
	.439	1.0052	.61	1.509
	.503	1.0253	.58	1.447
	.566	1.0426	.54	1.410
	.629	1.0570	.51	1.375
	.692	1.0685	.49	1.341
	.756	1.0800	.46	1.321
	.819	1.0901	.45	1.291
	.883	1.0987	.44	1.266
	.946	1.1074	.42	1.252
	1.010	1.1146	.40	1.240
Frame Factor 5	.251	.9317	.80	1.716
	.313	.9706	.72	1.602
	.376	.9994	.66	1.519
	.439	1.0224	.61	1.458
	.503	1.0411	.57	1.409
	.566	1.0570	.54	1.367
	.629	1.0699	.51	1.334
	.692	1.0814	.49	1.304
	.756	1.0916	.47	1.278
Frame Factor 6	.251	.9504	.82	1.642
	.313	.9864	.72	1.554
	.376	1.0138	.65	1.485
	.439	1.0354	.61	1.419
	.503	1.0541	.57	1.373
	.566	1.0685	.54	1.334
	.629	1.0814	.51	1.305
	.692	1.0916	.49	1.276
	.756	1.1016	.47	1.253
	.819	1.1102	.45	1.234
	.883	1.1175	.43	1.219
	.946	1.1246	.42	1.201
	1.010	1.1304	.40	1.191

TABLES OF MAXIMUM VALUES FROM WENKS FORMULA
HENCKY-VON MISES YIELD CRITERION-MIDPLANE, MIDBAY

	$100h/D_m$	η max/1.2	λ	ψ
Frame Factor 4	.376	1.1333	.82	1.646
	.439	1.164	.76	1.611
	.503	1.198	.72	1.562
	.566	1.214	.67	1.533
	.629	1.237	.63	1.506
	.692	1.248	.60	1.481
	.756	1.265	.58	1.454
	.819	1.270	.55	1.438
	.883	1.280	.52	1.425
	.946	1.292	.50	1.409
	1.010	1.303	.49	1.391
Frame Factor 5	.376	1.1592	.84	1.620
	.439	1.1880	.76	1.571
	.503	1.2139	.70	1.538
	.566	1.2341	.67	1.497
	.629	1.2500	.63	1.472
	.692	1.2644	.60	1.448
	.756	1.2759	.57	1.428
Frame Factor 6	.376	1.1794	.84	1.573
	.439	1.2068	.76	1.540
	.503	1.2298	.71	1.502
	.566	1.2471	.66	1.476
	.629	1.2609	.62	1.451
	.692	1.2744	.60	1.423
	.756	1.2859	.57	1.405
	.819	1.2960	.54	1.390
	.883	1.3032	.52	1.374
	.946	1.3118	.51	1.356
	1.010	1.3176	.49	1.345

FRAME PARAMETERS AT MAXIMUM EFFICIENCY #92A

	$100h/D_m$		I_1/I_F	100b/D
Frame Factor 4	.251	.80	4.758	.2195
	.313	.72	5.335	.2350
	.376	.66	5.860	.2485
	.439	.61	6.340	.2640
	.503	.58	6.817	.2720
	.566	.54	7.238	.2780
	.629	.51	7.655	.2850
	.692	.49	8.072	.2930
	.756	.46	8.464	.2960
	.819	.45	8.871	.3045
	.883	.44	9.267	.3120
	.946	.42	9.662	.3145
	1.010	.40	10.065	.3150
Frame Factor 5	.251	.80	5.064	.2065
	.313	.72	5.672	.2215
	.376	.66	6.224	.2340
	.439	.61	6.735	.2420
	.503	.57	7.212	.2525
	.566	.54	7.679	.2610
	.629	.51	8.124	.2675
	.692	.49	8.567	.2745
	.756	.47	9.001	.2810
Frame Factor 6	.251	.82	5.364	.1990
	.313	.72	5.958	.2110
	.376	.66	6.533	.2220
	.439	.61	7.062	.2315
	.503	.57	7.566	.2400
	.566	.54	8.056	.2475
	.629	.51	8.527	.2535
	.692	.49	8.995	.2605
	.756	.47	9.451	.2660
	.819	.45	9.919	.2700
	.883	.43	10.391	.2725
	.946	.42	10.853	.2785
	1.010	.40	11.354	.2790

FRAME PARAMETERS AT MAXIMUM EFFICIENCY #WENK

	$100h/D_m$		I_1/I_F	100b/D
Frame Factor 4	.376	.82	6.192	.2800
	.439	.76	6.647	.2925
	.503	.72	7.085	.3055
	.566	.67	7.468	.3140
	.629	.63	7.843	.3210
	.692	.60	8.213	.3290
	.756	.58	8.580	.3375
	.819	.55	8.930	.3420
	.883	.52	9.280	.3440
	.946	.50	9.634	.3495
	1.010	.49	9.986	.3555
Frame Factor 5	.376	.84	6.568	.2670
	.439	.76	7.008	.2755
	.503	.70	7.433	.2830
	.566	.67	7.878	.2950
	.629	.63	8.264	.3010
	.692	.60	8.657	.3090
	.756	.57	9.041	.3140
Frame Factor 6	.376	.84	6.853	.2540
	.439	.76	7.311	.2620
	.503	.71	7.768	.2715
	.566	.66	8.195	.2780
	.629	.62	8.616	.2835
	.692	.60	9.037	.2930
	.756	.57	9.446	.2980
	.819	.54	9.859	.3010
	.883	.52	10.273	.3060
	.946	.51	10.678	.3130
	1.010	.49	11.106	.3160

EXPERIMENTAL DATA FROM REFERENCE 9

Model Number	100h/D	$\sigma_y/1000$	λ	ψ_{92A}	ψ_{exp}
1	.88	46.8	.41	1.43	1.68
2	.784	47.5	.42	1.42	1.74
BR7m	.784	59.2	.46	1.44	1.62
BR7	.599	44.8	.59	1.11	1.3
5	.814	47.1	.61	1.01	1.24
6	.474	49.6	.67	1.09	1.24
7	.581	47.0	.70	.997	1.11

ψ_{exp} represents ψ determined using actual collapse pressure and ψ_{92A} was determined using collapse pressure calculated by formula #92A.

EXPERIMENTAL DATA FROM REFERENCE 19

General Model Parameters

Left/D_m = .1405

E = 30,000,000 lb/in^2

Light Frames

MODEL NUMBER	100h/D	σ_y/1000	λ	ψ_{exp}
79	.301	27	.876	.92
82	.303	28	.855	.90
76	.307	42	1.06	.66
88	.293	27	.890	.85
75	.294	39	1.06	.91
81	.295	28	.900	.93
84	.300	29	.909	.81
87	.300	29	.905	.81
80	.310	28	.840	.87
85	.311	29	.884	.83
92	.288	32	1.00	.85
91	.290	32	.975	.84
93	.308	29	.886	.88
96	.298	28	.896	.85
94	.298	28	.896	.84
90	.300	27	.88	.82

In the computation of λ values, h/D was used instead of h/D_m resulting in a small error which was not critical in the graphical methods used.

ψ_{exp} represents the ratio of actual collapse pressure to the hoop stress collapse pressure.

ALUMINAUT

In the following section, λ, ψ, and η have been found for the pressure hull in Aluminaut. The dimensions of that craft were taken from References (5) and (15) and are for the cylindrical hull alone.

Operating Depth	=	15,000 ft.	LOA	=	33.3 ft.
Safety Factor	=	1.4	L_e	=	6.6 ft.
σ_y	=	60,000 psi	D	=	8.0 ft.
E	=	29,000,000 psi	D_m	=	7.5 ft.
ρ	=	.1 lb/in^3	h	=	.5 ft.

1. $\lambda = \sqrt[4]{\frac{(L/D_m)^2}{(h/D_m)^3}} \sqrt{\frac{\sigma_y}{E}}$

$\lambda = .324$

2. $\psi = \frac{p_c}{2\frac{h}{D}\sigma_y}$ $\qquad p_c = 1.4 \times 15{,}000 \times .446$

$p_c = 9400$ psi

$\psi = 1.25$

3. $\eta = \frac{p_c V}{W\ 10^4}$

$\eta = 2.65$ in.

From the above computations, it is apparent that the designers of Aluminaut produced a vessel whose parameters put it in the low lambda design region. However, it has been noted (15) that a more efficient structure is possible.

Thesis 134735
H5264 Hickey
An investigation of low lambda yield failure design of stiffened cylindrical pressure vessels under hydrostatic pressure.

5 SEP 72 DISPLAY

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H5264 Hickey
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